Feedback Control Of Dynamic Systems (8th Edition)

Step-by-step solution

Step 1 of 7

Refer to Figure 6.85 in the textbook.

Observe that, there initial slope present in the magnitude plot. Thus, there must be at least one pole at the origin. To know the number of poles at origin, the slope must be known.

Calculate the initial slope, m.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-10}{10-1} = -1$$

Slope through one square block is 20 dB/decade.

Thus, in dB/decade, the initial slope of the provided curve is,

$$m_{\text{db/decade}} = m(20\text{ dB/decade}) = (-1)(20\text{ dB/decade}) = -20\text{ dB/decade}$$

Since the initial slope is -20 dB/decade, there is only one pole at the origin.

Therefore, one pole is present at $\omega = 0$.

Comment

Step 2 of 7

Observe from the Figure 6.85 in the textbook that at $\omega = 10$, the slope changes to 0 dB/decade.

Recall that a zero introduces a slope change by $+20\text{ dB/decade}$.

Thus, a zero is present in the system at $\omega = 10$ such that,

$$-20\text{ dB/decade} + 20\text{ dB/decade} = 0\text{ dB/decade}$$

Comment

Step 3 of 7

For the known zeros and poles, write the transfer function as,

$$G(s) = \frac{K\left(1 + \frac{s}{10}\right)}{s}$$

Put $s = j\omega$.

$$G(j\omega) = \frac{K\left(1 + \frac{j\omega}{10}\right)}{j\omega}$$

Calculate the magnitude of $G(j\omega)$.

$$|G| = \frac{K\sqrt{1 + \frac{\omega^2}{100}}}{\omega}$$

$$10 = K\sqrt{\frac{101}{100}}$$
$$K \approx 10$$

Comment

Step 4 of 7

Thus, the transfer function is,

$$G(s) = \frac{10\left(1+\frac{s}{10}\right)}{s} = \frac{s+10}{s}$$

Consider $Y(s)$ as the output and $R(s)$ as the input.

Recall that $G(s) = \frac{Y(s)}{R(s)}$ Thus,

$$\frac{Y(s)}{R(s)} = \frac{s+10}{s}$$
$$Y(s) = \left(\frac{s+10}{s}\right)R(s) \quad \text{...... (1)}$$

Comment

Step 5 of 7

Consider unit step input, i.e., $r(t) = u(t)$.

Thus, $R(s) = \frac{1}{s}$.

Substitute $\frac{1}{s}$ for $R(s)$ in (1).

$$Y(s) = \left(\frac{s+10}{s}\right)\left(\frac{1}{s}\right)$$
$$Y(s) = \frac{1}{s} + \frac{10}{s^2}$$

Take inverse Laplace both the sides.

$$y(t) = u(t) + 10tu(t) = (1+10t)u(t)$$

Comment

Step 6 of 7

Chapter 6, Problem 9P

$$y(t)=\begin{cases}10t+1 & for\ t>0\\ 0 & for\ t<0\end{cases}$$

Recall the equation of straight line equation, $y=ax+c$.

Here, a is the slope of the line and c is the intercept made on y-axis.

Thus, the slope of the line curve of $y(t)$ will be 10 and the intercept made on y-axis will be 1.

Comment

Step 7 of 7

Sketch the straight line for $y(t)$ with 10 as slope as shown in Figure 1.

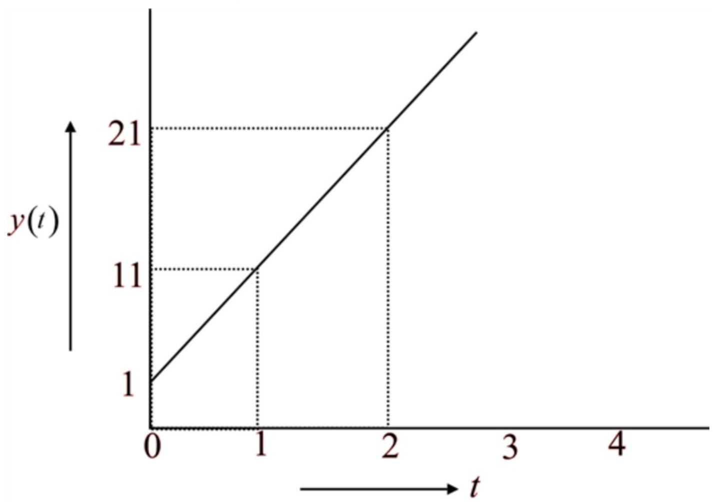

Figure 1

Therefore, the plot for unit-step response of the system is sketched.

Comment

Recommended solutions for you in Chapter 6