Feedback Control Of Dynamic Systems (8th Edition)

Step-by-step solution

Step 1 of 23

Refer to control system in Figure 5.45 in the textbook.

Consider the open loop transfer function.

$$G(s)=\frac{5}{s(s+2)}$$

And for the unity feedback system,

$$H(s)=1+\alpha s$$

Write the characteristic equation.

$$1+G(s)H(s)=0$$

$$1+\left(\frac{5}{s(s+2)}\right)(1+\alpha s)=0$$

$$s(s+2)+5(1+\alpha s)=0$$

$$s^2+2s+5+5\alpha s=0$$

$$1+\frac{5\alpha s}{s^2+2s+5}=0 \quad \ldots\ldots (1)$$

The above equation is in the form,

$$1+K\frac{b(s)}{a(s)}=0 \quad \ldots\ldots (2)$$

Comment

Step 2 of 23

Equate equation (1) and (2).

$$a(s)=s^2+2s+5$$
$$b(s)=5s$$

The open loop transfer function is,

$$L(s)=\frac{b(s)}{a(s)}$$
$$=\frac{5s}{s^2+2s+5}$$

Therefore, the values of $L(s)$, $a(s)$ and $b(s)$ are,

$$\boxed{\begin{aligned}L(s)&=\frac{5s}{s^2+2s+5}\\a(s)&=s^2+2s+5\\b(s)&=5s\end{aligned}}.$$

Comment

Step 3 of 23

MATLAB code to plot the root locus:

```
sys=tf(num,den);
rlocus(sys)
```

Comment

Step 4 of 23

Consider the following root locus plot as shown in Figure 1.

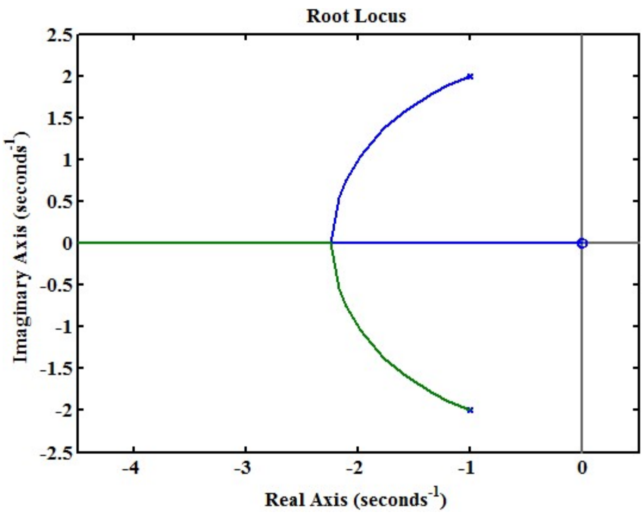

Figure 1: Root locus

Comment

Step 5 of 23

Calculate the closed loop transfer function.

$$T(s)=\frac{\frac{5}{s(s+2)}}{1+(1+\alpha s)\left(\frac{5}{s(s+2)}\right)}$$

$$=\frac{5}{s(s+2)+5+5\alpha s}$$

$$=\frac{5}{s^2+2s+5+5\alpha s}$$

$$=\frac{5}{s^2+(2+5\alpha)s+5}$$

Comment

Step 6 of 23

Calculate the closed loop pole locations when $\alpha=0$.

$$s^2+(2+5\alpha)s+5$$

$$s^2+[2+5(0)]s+5$$

$$s^2+2s+5=0$$

$$s_{1,2}=-1\pm j2$$

Therefore, the closed loop pole locations when $\alpha=0$ are $\boxed{-1\pm j2}$.

Comment

Write the closed loop transfer function.

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2+2s+5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2+2s+5)}$$

Calculate the step response when $\alpha = 0$.

$$\begin{aligned} Y(s) &= \frac{5}{s(s^2+2s+5)} \\ &= \frac{1}{s} - \frac{s+2}{(s+1)^2+2^2} \\ &= \frac{1}{s} - \frac{s+1}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+2^2} \end{aligned}$$

Apply inverse Laplace transform.

$$y(t) = u(t) - e^{-t}\cos(2t)u(t) - 0.5e^{-t}\sin(2t)u(t)$$

Comment

Step 8 of 23

MATLAB code to plot the step response:

```
t=0:0.01:5;
y=1-exp(-t).*cos(2.*t)-0.5.*exp(-t).*sin(2.*t);
plot(t,y)
title('step response when a=0');
```

Comment

Step 9 of 23

Consider the following step response as shown in Figure 2.

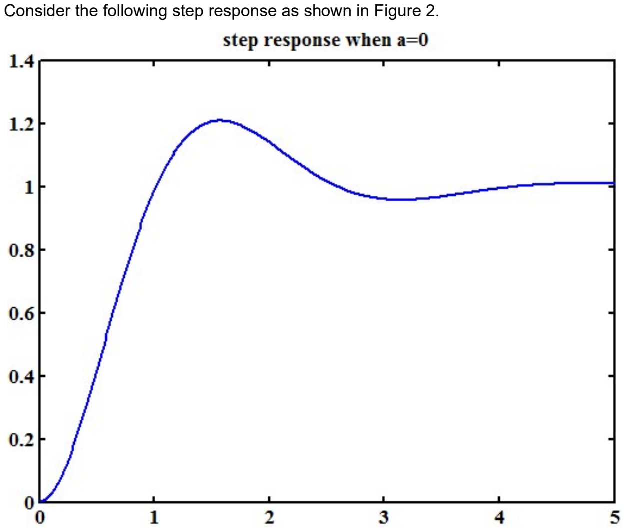

Figure 2

Comment

Step 10 of 23

MATLAB code to plot step response when $\alpha = 0$:

```
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=0');
```

Comment

Step 11 of 23

Consider the following step response when $\alpha = 0$ as shown in Figure 3.

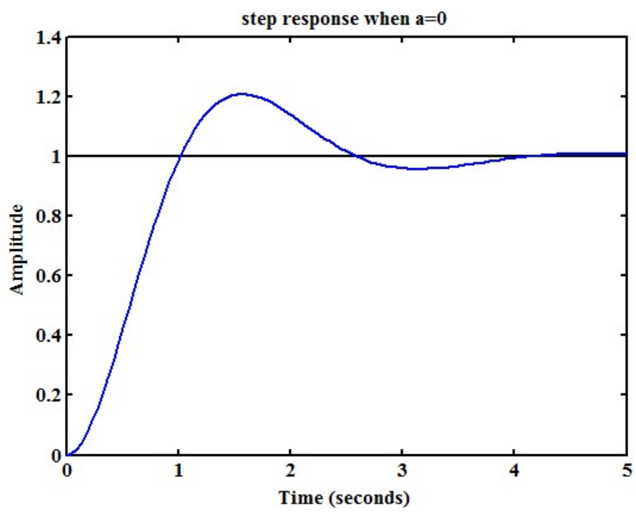

Figure 3

Thus, the step responses in Figure 2 and Figure 3 are same.

Comment

Step 12 of 23

Calculate the closed loop pole locations when $\alpha = 0.5$.

$$s^2 + (2+5\alpha)s + 5$$
$$s^2 + [2+5(0.5)]s + 5$$
$$s^2 + 4.5s + 5 = 0$$
$$s_{1,2} = -2, -2.5$$

Therefore, the closed loop pole locations when $\alpha = 0.5$ are $\boxed{-2, -2.5}$.

Comment

Step 13 of 23

The closed loop transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + 4.5s + 5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2 + 4.5s + 5)}$$

Calculate the step response when $\alpha = 0$.

$$= \frac{\;}{s} - \frac{\;}{s+2} + \frac{\;}{s+2.5}$$

Apply inverse Laplace transform.

$$y(t) = u(t) - 5e^{-2t}u(t) + 4e^{-2.5t}u(t)$$
$$= \left[1 - 5e^{-2t} + 4e^{-2.5t}\right]u(t)$$

Comment

Step 14 of 23

MATLAB code to plot the step response:

```
t=0:0.01:5;
y=1-5.*exp(-2.*t)+4.*exp(-2.5.*t);
plot(t,y)
title('step response when a=0.5');
```

Comment

Step 15 of 23

The step response is shown in Figure 4.

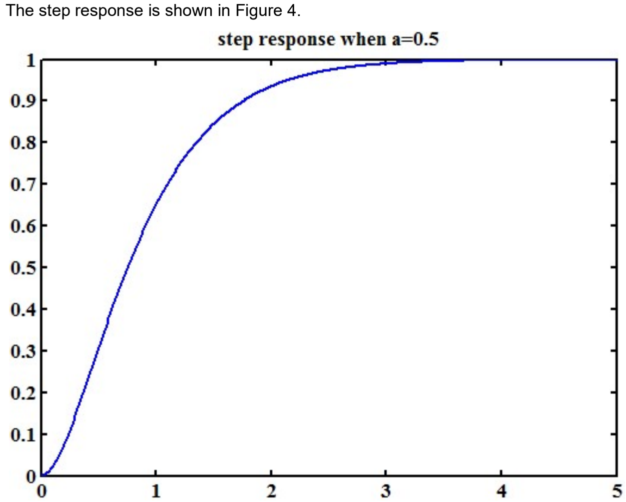

Figure 4

Comment

Step 16 of 23

MATLAB code to plot step response when $\alpha = 0.5$:

```
a=0.5;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=0.5');
```

Comment

Step 17 of 23

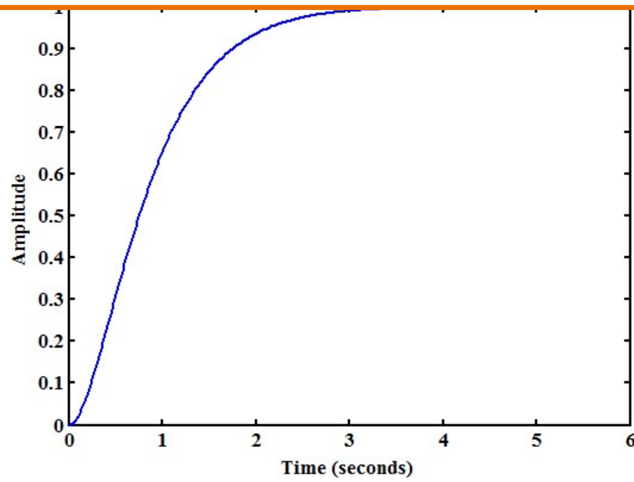

Figure 5

Thus, the step responses in Figure 4 and Figure 5 are same.

Comment

Step 18 of 23

Calculate the closed loop pole locations when $\alpha = 2$.

$$s^2 + (2+5\alpha)s + 5$$
$$s^2 + [2+5(2)]s + 5$$
$$s^2 + 12s + 5 = 0$$
$$s_{1,2} = -0.432, -11.5567$$

Therefore, the closed loop pole locations when $\alpha = 2$ are $\boxed{-0.432, -11.5567}$.

Comment

Step 19 of 23

The closed loop transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + 12s + 5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2 + 12s + 5)}$$

Calculate the step response when $\alpha = 0$.

$$Y(s) = \frac{5}{s(s+0.432)(s+11.5567)}$$
$$= \frac{1.0015}{s} - \frac{1.0404}{s+0.432} + \frac{0.039}{s+11.5567}$$

Apply inverse Laplace transform.

$$y(t) = 1.0015u(t) - 1.0404e^{-0.432t}u(t) + 0.039e^{-11.5567t}u(t)$$
$$= \left[1.0015 - 1.0404e^{-0.432t} + 0.039e^{-11.5567t}\right]u(t)$$

Comment

Step 20 of 23

MATLAB code to plot the step response:

```
plot(t,y)
title('step response when a=2');
```

Comment

Step 21 of 23

Consider the following step response as shown in Figure 6.

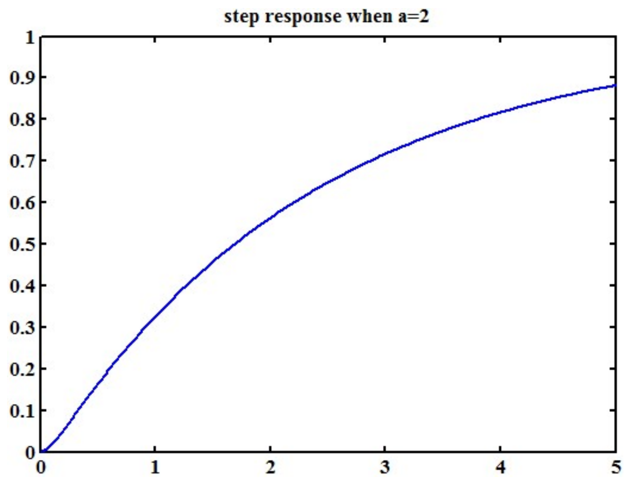

Figure 6

Comment

Step 22 of 23

MATLAB code to plot step response when $\alpha = 2$:

```
a=2;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=2');
```

Comment

Step 23 of 23

Consider the following step response when $\alpha = 2$ as shown in Figure 7.

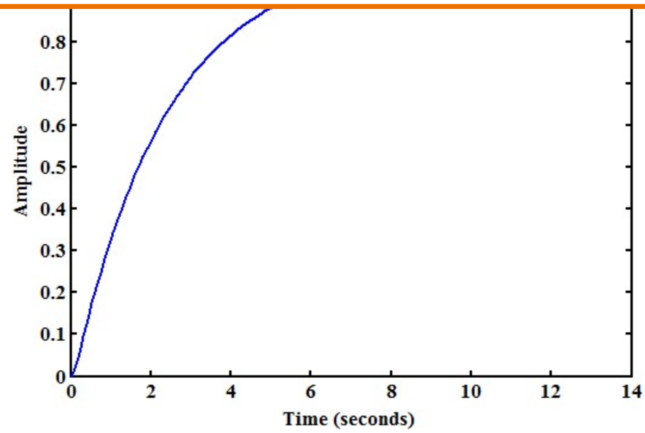

Figure 7

Thus, on comparing the step responses shown in Figure 6 with Figure 7 pointwise, both the responses are same.