

Feedback Control Of Dynamic Systems | (8th Edition)

Chapter 5, Problem 9P

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Step-by-step solution

Step 1 of 23

Refer to control system in Figure 5.45 in the textbook.

Consider the open loop transfer function.

$$G(s) = \frac{5}{s(s+2)}$$

And for the unity feedback system,

$$H(s) = 1 + \alpha s$$

Write the characteristic equation.

$$1 + G(s)H(s) = 0$$

$$1 + \left(\frac{5}{s(s+2)} \right) (1 + \alpha s) = 0$$

$$s(s+2) + 5(1 + \alpha s) = 0$$

$$s^2 + 2s + 5 + 5\alpha s = 0$$

$$1 + \frac{5\alpha s}{s^2 + 2s + 5} = 0 \dots\dots (1)$$

The above equation is in the form,

$$1 + K \frac{b(s)}{a(s)} = 0 \dots\dots (2)$$

[Comment](#)

Step 2 of 23

Equate equation (1) and (2).

$$a(s) = s^2 + 2s + 5$$

$$b(s) = 5s$$

The open loop transfer function is,

$$L(s) = \frac{b(s)}{a(s)} \\ = \frac{5s}{s^2 + 2s + 5}$$

Therefore, the values of $L(s)$, $a(s)$ and $b(s)$ are,

$$\boxed{\begin{array}{l} L(s) = \frac{5s}{s^2 + 2s + 5} \\ a(s) = s^2 + 2s + 5 \\ b(s) = 5s \end{array}}$$

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Step 3 of 23

MATLAB code to plot the root locus:

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
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```
sys=tf(num,den);
rlocus(sys)
```

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Step 4 of 23

Consider the following root locus plot as shown in Figure 1.

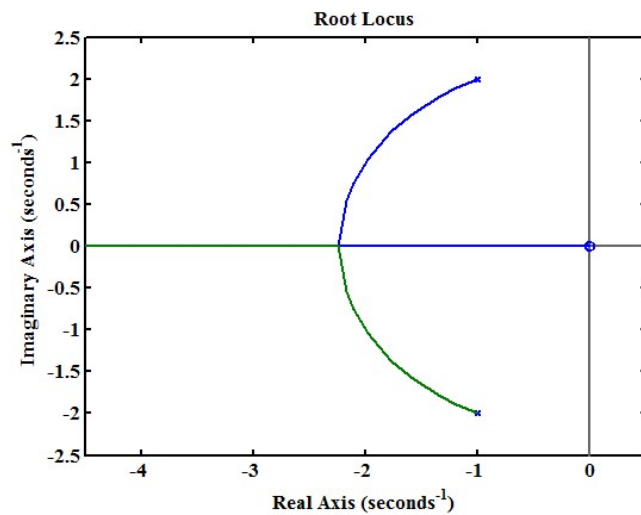


Figure 1: Root locus

[Comment](#)

Step 5 of 23

Calculate the closed loop transfer function.

$$\begin{aligned}
 T(s) &= \frac{\frac{5}{s(s+2)}}{1 + (1+\alpha s)\left(\frac{5}{s(s+2)}\right)} \\
 &= \frac{5}{s(s+2) + 5 + 5\alpha s} \\
 &= \frac{5}{s^2 + 2s + 5 + 5\alpha s} \\
 &= \frac{5}{s^2 + (2+5\alpha)s + 5}
 \end{aligned}$$

[Comment](#)

Step 6 of 23

Calculate the closed loop pole locations when $\alpha = 0$.

$$\begin{aligned}
 &s^2 + (2+5\alpha)s + 5 \\
 &s^2 + [2+5(0)]s + 5 \\
 &s^2 + 2s + 5 = 0 \\
 &s_{1,2} = -1 \pm j2
 \end{aligned}$$

Therefore, the closed loop pole locations when $\alpha = 0$ are $\boxed{-1 \pm j2}$.

[Comment](#)

Write the closed loop transfer function.

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + 2s + 5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2 + 2s + 5)}$$

Calculate the step response when $\alpha = 0$.

$$\begin{aligned} Y(s) &= \frac{5}{s(s^2 + 2s + 5)} \\ &= \frac{1}{s} - \frac{s+2}{(s+1)^2 + 2^2} \\ &= \frac{1}{s} - \frac{s+1}{(s+1)^2 + 2^2} - \frac{1}{(s+1)^2 + 2^2} \end{aligned}$$

Apply inverse Laplace transform.

$$y(t) = u(t) - e^{-t} \cos(2t)u(t) - 0.5e^{-t} \sin(2t)u(t)$$

[Comment](#)

Step 8 of 23

MATLAB code to plot the step response:

```
t=0:0.01:5;
y=1-exp(-t).*cos(2.*t)-0.5.*exp(-t).*sin(2.*t);
plot(t,y)
title('step response when a=0');
```

[Comment](#)

Step 9 of 23

Consider the following step response as shown in Figure 2.

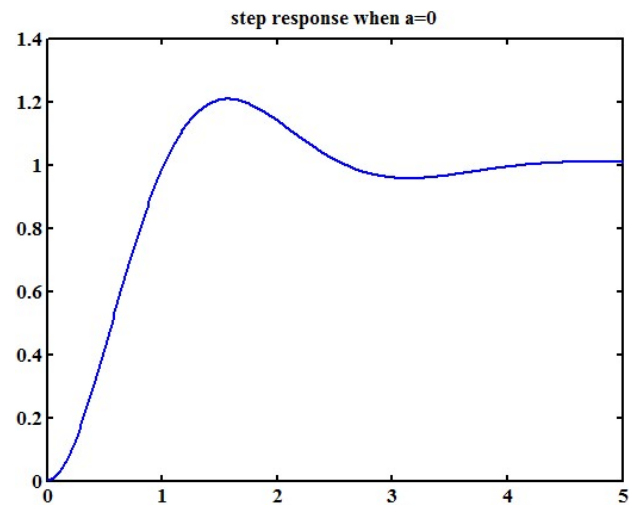


Figure 2

[Comment](#)

Step 10 of 23

MATLAB code to plot step response when $\alpha = 0$:

```
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=0');
```

[Comment](#)

Step 11 of 23

Consider the following step response when $\alpha = 0$ as shown in Figure 3.

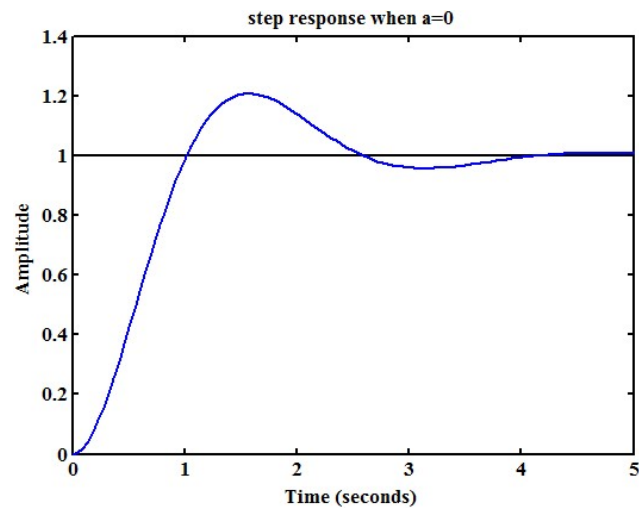


Figure 3

Thus, the step responses in Figure 2 and Figure 3 are same.

[Comment](#)

Step 12 of 23

Calculate the closed loop pole locations when $\alpha = 0.5$.

$$s^2 + (2 + 5\alpha)s + 5$$

$$s^2 + [2 + 5(0.5)]s + 5$$

$$s^2 + 4.5s + 5 = 0$$

$$s_{1,2} = -2, -2.5$$

Therefore, the closed loop pole locations when $\alpha = 0.5$ are $\boxed{-2, -2.5}$.

[Comment](#)

Step 13 of 23

The closed loop transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + 4.5s + 5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2 + 4.5s + 5)}$$

Calculate the step response when $\alpha = 0$.

$$= \frac{1}{s} - \frac{1}{s+2} + \frac{1}{s+2.5}$$

Apply inverse Laplace transform.

$$y(t) = u(t) - 5e^{-2t}u(t) + 4e^{-2.5t}u(t)$$

$$= [1 - 5e^{-2t} + 4e^{-2.5t}]u(t)$$

[Comment](#)

Step 14 of 23

MATLAB code to plot the step response:

```
t=0:0.01:5;
y=1-5.*exp(-2.*t)+4.*exp(-2.5.*t);
plot(t,y)
title('step response when a=0.5');
```

[Comment](#)

Step 15 of 23

The step response is shown in Figure 4.

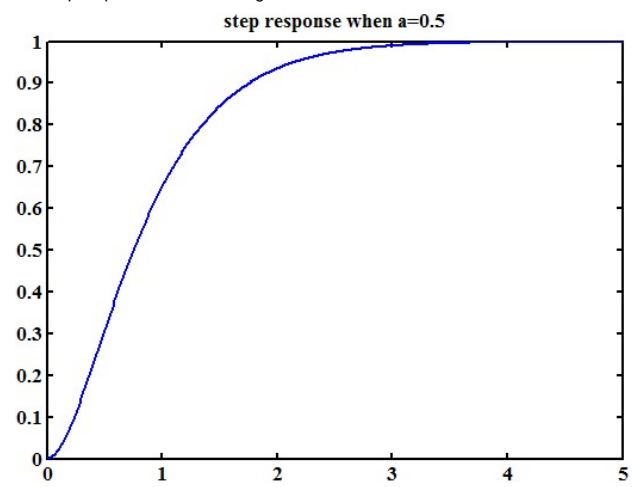


Figure 4

[Comment](#)

Step 16 of 23

MATLAB code to plot step response when $\alpha = 0.5$:

```
a=0.5;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=0.5');
```

[Comment](#)

Step 17 of 23

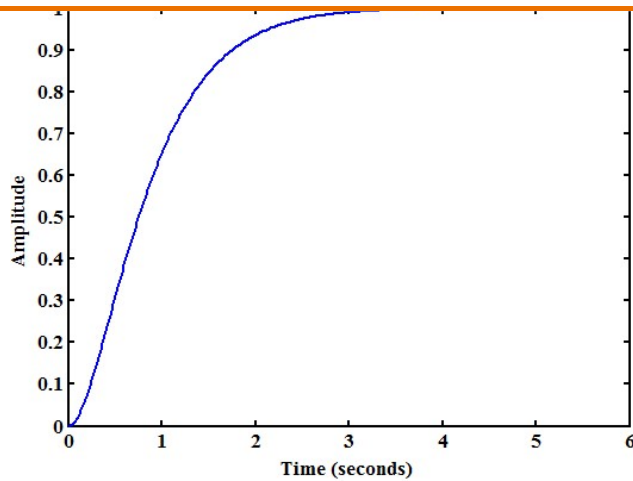


Figure 5

Thus, the step responses in Figure 4 and Figure 5 are same.

[Comment](#)

Step 18 of 23

Calculate the closed loop pole locations when $\alpha = 2$.

$$s^2 + (2 + 5\alpha)s + 5$$

$$s^2 + [2 + 5(2)]s + 5$$

$$s^2 + 12s + 5 = 0$$

$$s_{1,2} = -0.432, -11.5567$$

Therefore, the closed loop pole locations when $\alpha = 2$ are $\boxed{-0.432, -11.5567}$.

[Comment](#)

Step 19 of 23

The closed loop transfer function is,

$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + 12s + 5}$$

Substitute $\frac{1}{s}$ for $R(s)$ in the above equation.

$$Y(s) = \frac{5}{s(s^2 + 12s + 5)}$$

Calculate the step response when $\alpha = 0$.

$$\begin{aligned} Y(s) &= \frac{5}{s(s + 0.432)(s + 11.5567)} \\ &= \frac{1.0015}{s} - \frac{1.0404}{s + 0.432} + \frac{0.039}{s + 11.5567} \end{aligned}$$

Apply inverse Laplace transform.

$$\begin{aligned} y(t) &= 1.0015u(t) - 1.0404e^{-0.432t}u(t) + 0.039e^{-11.5567t}u(t) \\ &= [1.0015 - 1.0404e^{-0.432t} + 0.039e^{-11.5567t}]u(t) \end{aligned}$$

[Comment](#)

Step 20 of 23

MATLAB code to plot the step response:

```

plot(t,y)
title('step response when a=2');

```

[Comment](#)

Step 21 of 23

Consider the following step response as shown in Figure 6.

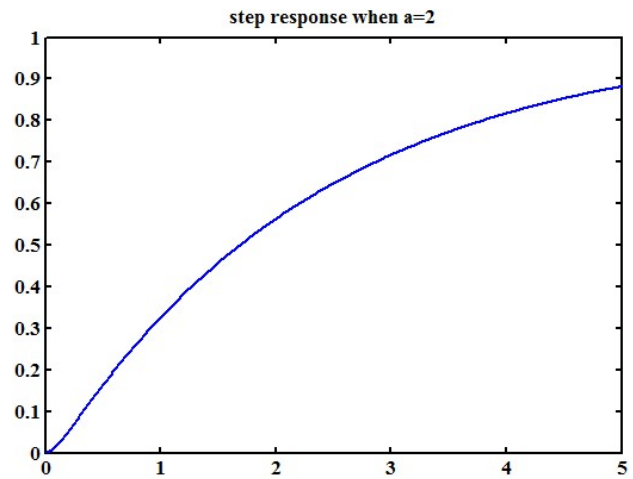


Figure 6

[Comment](#)

Step 22 of 23

MATLAB code to plot step response when $\alpha = 2$:

```

a=2;
num=[5];
den=[1 2+5*a 5];
sys=tf(num,den);
step(sys);
title('step response when a=2');

```

[Comment](#)

Step 23 of 23

Consider the following step response when $\alpha = 2$ as shown in Figure 7.

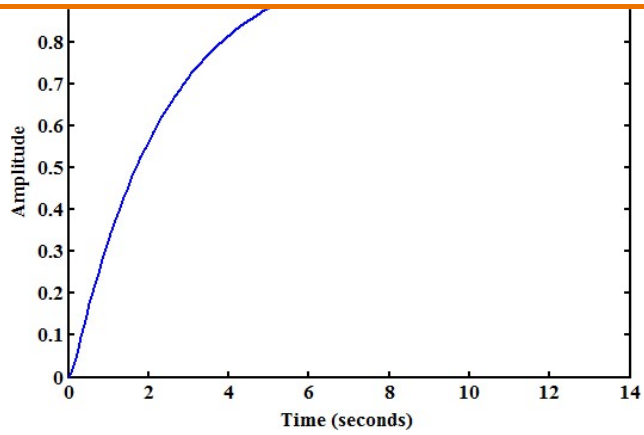


Figure 7

Thus, on comparing the step responses shown in Figure 6 with Figure 7 pointwise, both the responses are same.

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